

Extraction of Neutrino Flux with the Low ν Method at MiniBooNE Energies

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Abstract. We describe the application of the ‘low- ν ’ method to the extraction of the neutrino flux at MiniBooNE energies. As an example, we extract the relative energy dependence of the flux from published MiniBooNE quasielastic scattering cross sections with $\nu < 0.2$ GeV and $\nu < 0.1$ GeV (here ν is the energy transfer to the target). We find that the flux extracted from the ‘low- ν ’ cross sections is consistent with the nominal flux used by MiniBooNE. We fit the MiniBooNE cross sections over the entire kinematic range to various parametrizations of the axial form factor. We find that if the overall normalization of the fit is allowed to float within the normalization errors, the extracted values of the axial vector mass are independent of the flux. Within the Fermi gas model, the Q^2 distribution of the MiniBooNE data is described by a standard dipole form factor with $M_A = 1.41 \pm 0.04$ GeV. If nuclear transverse enhancement in the vector form factors is accounted for, the data are best fit with a modified dipole form factor with $M_A = 1.10 \pm 0.03$ GeV.

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In a previous communication [1] we present the application of the ‘low- ν ’ method to the extraction of neutrino (ν_μ) flux for energies (E_ν) as low as 0.7 GeV. In this paper we extend the technique to E_ν as low as 0.4 GeV and extract the relative energy dependence of the ν_μ flux for the MiniBooNE experiment as an example.

The charged current ν_μ ($\bar{\nu}_\mu$) differential cross section can be written in terms of the square of the four momentum (Q^2) and energy transfer (ν) to the target nucleus. At low- ν , if we integrate the cross section from $\nu_{min} \approx 0$ up to $\nu = \nu_{cut}$ (where ν_{cut} is small), we can write the ‘low- ν ’ cross section [1] in terms of an energy independent term which is proportional to the structure function \mathcal{W}_2 , and small energy dependent corrections which are proportional to ν/E , or m_μ^2/E^2 where $m - \mu$ is the mass of the muon.

$$\sigma_{\nu_{cut}}(E_\nu) = \int_{\nu_{min}(E_\nu)}^{\nu_{cut}} \frac{d^2\sigma}{dQ^2 d\nu} dQ^2 d\nu = \sigma_{W_2} + \sigma_2 + \sigma_1 \pm \sigma_3 + \sigma_4 + \sigma_5$$

$$\sigma_{W_2} = C \int_{\nu_{min}(E_\nu)}^{\nu_{cut}} \mathcal{W}_2 d\nu; \quad \sigma_2 = C \int_{\nu_{min}(E_\nu)}^{\nu_{cut}} \left[-\frac{\nu}{E_\nu} - \frac{Q^2 + m_\mu^2}{4E_\nu^2} \right] \mathcal{W}_2 d\nu$$

$$\sigma_1 = C \int_{\nu_{min}(E_\nu)}^{\nu_{cut}} \frac{(Q^2 + m_\mu^2)}{2E_\nu^2} \mathcal{W}_1 d\nu; \quad \sigma_3 = C \int_{\nu_{min}(E_\nu)}^{\nu_{cut}} \left[\frac{Q^2}{2ME_\nu} - \frac{\nu}{4E_\nu} \frac{Q^2 + m_\mu^2}{ME_\nu} \right] \mathcal{W}_3 d\nu,$$

where σ_4 and σ_5 are negligible[1], and M is the nucleon mass. The uncertainties in the modeling of the small energy dependent correction terms are small, thus we can extract the relative ν_μ flux from the number of low- ν events at each E_ν bin.

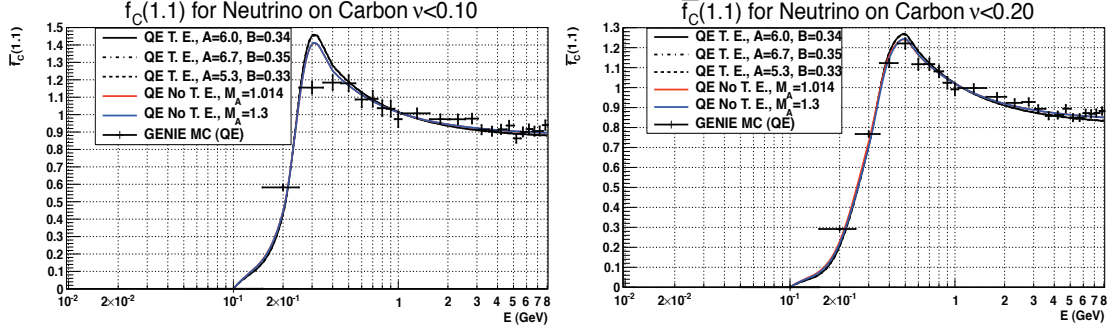


FIGURE 1. The ratio of the neutrino ‘low- ν ’ QE cross section (as a function of E_ν) to the ‘low- ν ’ QE cross section at $E_\nu = 1.1$ GeV for $\nu < 0.1$ GeV (left) and $\nu < 0.2$ GeV (right).

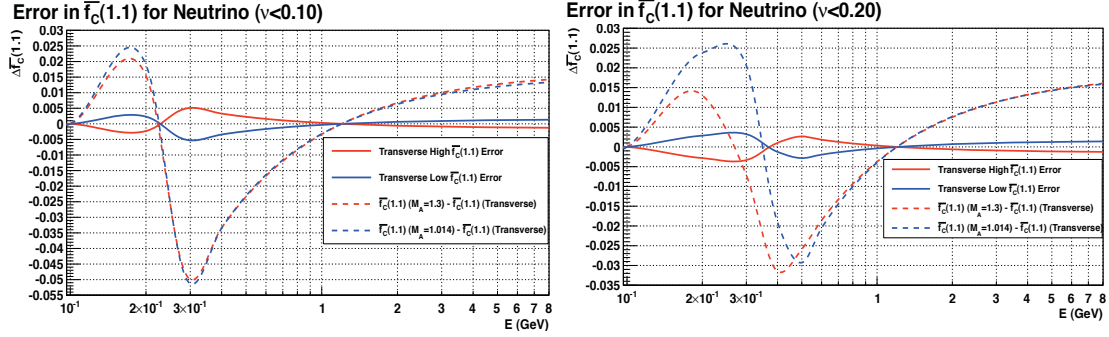


FIGURE 2. The model uncertainty in the ratio of the neutrino ‘low- ν ’ QE cross section (as a function of E_ν) to the ‘low- ν ’ QE cross section at $E_\nu = 1.1$ GeV for $\nu < 0.1$ GeV (left) and $\nu < 0.2$ GeV (right).

If we use the MINOS criteria that the fraction of events in the ‘low- ν ’ flux sample is lower than 60% of the total number of events in each $E_{\nu, \bar{\nu}_\mu}$ bin, we find that for neutrinos we can use events with $\nu < 0.1$ GeV to extract the relative flux for $E_\nu > 0.4$ GeV, and events with $\nu < 0.2$ GeV for $E_\nu > 0.7$ GeV. For these ν cuts, the cross section is dominated by quasielastic (QE) scattering. The flux extracted with the ‘low- ν ’ method is only a relative flux as a function of energy. It must be normalized at some energy. In this paper, we present the flux relative to the flux at $E_\nu = 1.1$ GeV. In our calculation of QE cross sections we use BBBA2007 electromagnetic form factors[3].

Figures 1 (ν_μ) and 3 ($\bar{\nu}_\mu$) show the ratio of the ‘low- ν ’ QE cross section (as a function of $E_{\nu, \bar{\nu}_\mu}$) to the ‘low- ν ’ cross section at $E_{\nu, \bar{\nu}_\mu} = 1.1$ GeV for $\nu < 0.1$ GeV (left) and $\nu < 0.2$ GeV (right) for various models. The data points are from the GENIE MC generator for a carbon target assuming a Fermi gas model and a dipole form for the axial form factor with $M_A = 0.99$ GeV[4]. The ratio is independent of the value M_A as illustrated by the fact that prediction of this ratio for a dipole axial vector mass $M_A = 1.014$ GeV (solid red line) and $M_A = 1.3$ GeV (solid blue line) are the same. Also shown are the changes in the prediction when we include nuclear enhancement in the transverse vector form factors[2] (TE) (shown as the solid black line). For $E_\nu > 0.4$ GeV and $\nu < 0.1$ and for $E_\nu > 0.7$ GeV and $\nu < 0.2$ the ratio is approximately constant.

The uncertainties in the modeling of the energy dependence of this ratio are small as shown in figures 2 and 4. As an example, we extract the relative energy dependence of

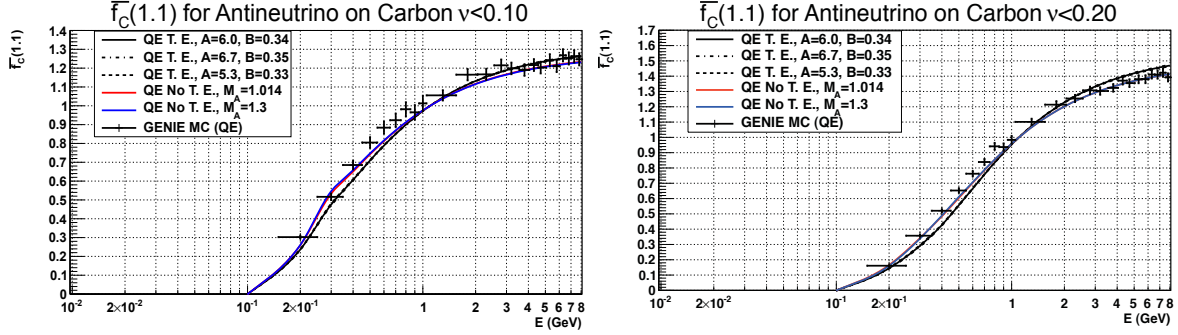


FIGURE 3. The ratio of the $\bar{\nu}_\mu$ ‘low- v ’ QE cross section (as a function of $E_{\bar{\nu}_\mu}$) to the ‘low- v ’ QE cross section at $E_{\bar{\nu}_\mu} = 1.1$ GeV for $v < 0.1$ GeV (left) and $v < 0.2$ GeV (right).

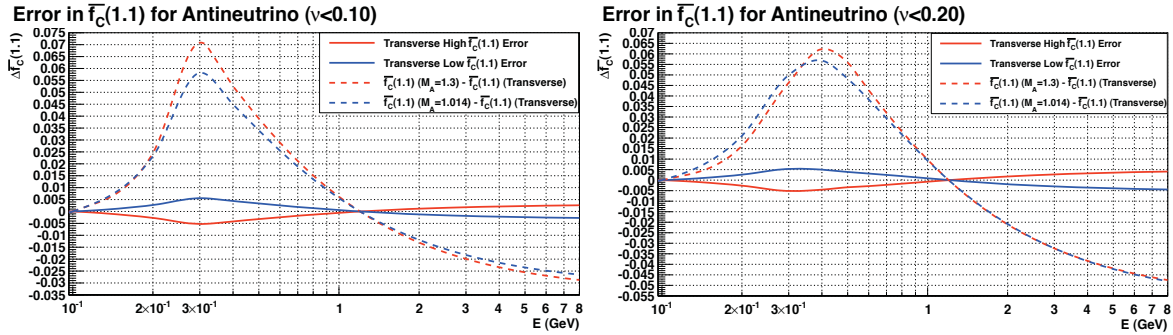


FIGURE 4. The model uncertainty in the ratio of the $\bar{\nu}_\mu$ ‘low- v ’ QE cross section (as a function of $E_{\bar{\nu}_\mu}$) to the ‘low- v ’ QE cross section at $E_{\bar{\nu}_\mu} = 1.1$ GeV for $v < 0.1$ GeV (left) and $v < 0.2$ GeV (right).

the flux of the Fermilab booster neutrino beam from MiniBooNE data. The MiniBooNE experiment published[6] flux weighted double differential cross sections for QE neutrino scattering in bins of final state muon kinetic energy T_μ ($E_\mu = T_\mu + m_\mu$) and muon angle ($\cos \theta_\mu$). We extract the central value of $v^{QE} = E_V^{QE} - E_\mu$ for each $(T_\mu, \cos \theta_\mu)$ bin using

$$E_V^{QE} = \frac{2(M'_n)E_\mu - ((M'_n)^2 + m_\mu^2 - M_p^2)}{2 \cdot [(M'_n) - E_\mu + \sqrt{E_\mu^2 - m_\mu^2 \cos \theta_\mu}]}. \quad (1)$$

where M_n and M_p are the neutron and proton mass, and $M'_n = M_n - E_B$ ($E_B = 34$ MeV).

The left side of Fig. 5 shows the MiniBooNE bins of 0.1 GeV in T_μ and 0.1 in $\cos \theta_\mu$. The solid lines are lines of constant E_V^{QE} . The blue and red dotted lines are $v^{QE} < 0.2$ GeV, and $v^{QE} < 0.1$ GeV, respectively. We extract the ‘low- v ’ flux from the MiniBooNE data as follows. Using the published MiniBooNE flux, we first fit the flux-weighted doubly differential cross section to three models. The parameters which are allowed to float within the models are the overall normalization and the axial vector mass M_A . The first model is a Fermi gas model with BBBA2007 electromagnetic form factors and a dipole form for the axial form factor. The second model includes Transverse

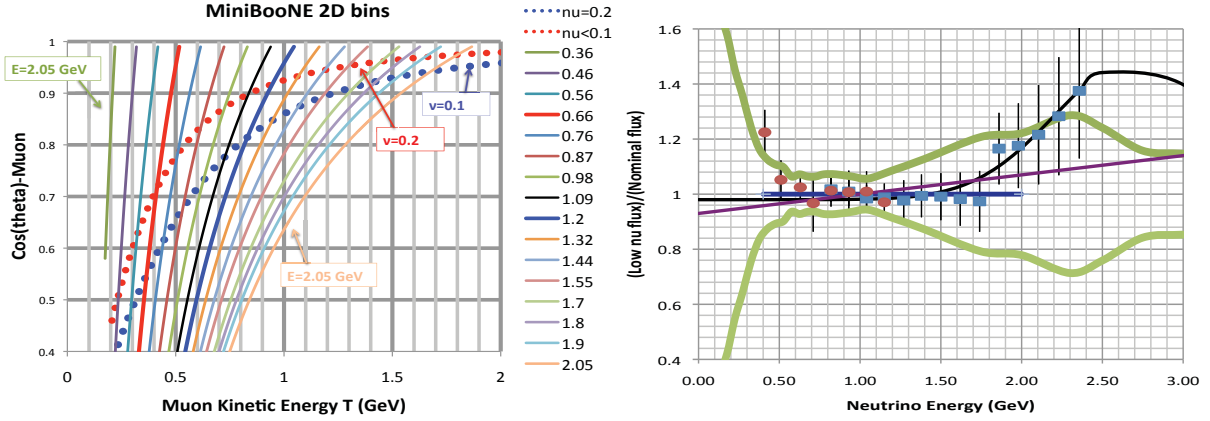


FIGURE 5. Left: The MiniBooNE QE cross section bins of 0.1 in $\cos \theta_\mu$, and 0.1 GeV in T_μ . The solid lines are lines of constant Q^E . The blue and red dotted lines are $\nu^{QE} < 0.2$ GeV, and $\nu^{QE} < 0.1$ GeV, respectively. Right: The relative neutrino flux extracted from $\nu^{QE} < 0.2$ GeV cross sections (blue squares) and $\nu^{QE} < 0.1$ GeV (red squares) shown with statistical errors only. The black (flux A) and purple (flux B) lines are possible deviations (which are consistent with the ‘low- ν ’ flux) from the central values of the published flux. The green line is the quoted systematic uncertainty in the nominal MiniBooNE flux.

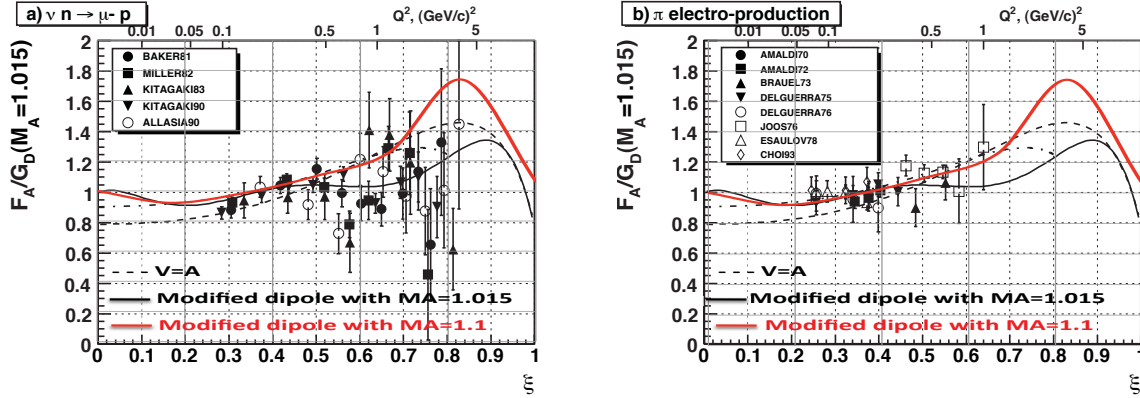


FIGURE 6. $F_A(Q^2)$ measurements on free nucleons (a) $F_A(Q^2)$ re-extracted from neutrino-deuteron data divided by $G_D^A(Q^2)$ (with $M_A = 1.015$ GeV). (b) $F_A(Q^2)$ from pion electroproduction divided by $G_D^A(Q^2)$ corrected for hadronic effects[7]. Solid black line - duality based modified dipole fit with $M_A = 1.015$ GeV[3]. Short-dashed line - $F_A(Q^2)_{A2=V2}$. Dashed-dot line - constituent quark model[8]. Solid red line - duality based modified dipole with $M_A = 1.10$ GeV, which is our best fit to the MiniBooNE data on Carbon (accounting for Transverse Enhancement).

Enhancement for the vector form factors[2] (BBC-TE) and a dipole form for the axial form factor. The third model includes TE for the vector form factors[2] (BBC-TE) and assumes a modified dipole form for the axial form factor as given in ref. [3]. The modification to the dipole form factor are from a fit[3] to all neutrino scattering data and pion electroproduction on free (H and D targets) nucleons as shown in fig. 6. The fit has the duality constraint that the vector and axial parts of structure function W_2 for quasielastic scattering are equal at large Q^2 .

The ratio of the flux-weighted MiniBooNE measured cross sections at low- ν to the calculated (with the nominal published MiniBooNE flux) flux-weighted cross sections for any of the three models is proportional to the ratio of the ‘low- ν ’ flux to the nominal MiniBooNE flux. As expected, the relative flux extracted as a function of neutrino energy is insensitive to the choice of model.

The left side of Fig. 5 shows the ratio of the flux extracted from $\nu < 0.1$ GeV events (red circles) and $\nu < 0.2$ GeV events (blue squares) to the nominal flux. Only statistical errors are shown. The green line is the systematic error in nominal flux (as published by MiniBooNE). The extracted ‘low- ν ’ flux is consistent with the nominal flux within the quoted systematic errors. The black curve (flux A) and purple curve (flux B) are possible deviations (which are consistent with the ‘low- ν ’ flux) from the nominal flux.

Next we fit for the best value of M_A for each of the three models. We find that if we let the overall normalization float within the systematic error the extracted values of M_A using the nominal flux, flux A, and flux B are within 0.015 GeV of each other as shown in Table 1. We find that with Transverse Enhancement, and a modified dipole form factor, the fit to the Q^2 dependence of the MiniBooNE $d\sigma/dQ^2$ on carbon favors an axial mass $M_A = 1.10 \pm 0.02$ GeV. The ratio of this modified dipole fit with $M_A = 1.10$ GeV to the simple dipole parametrization with $M_A = 1.015$ GeV is shown as the solid red line in fig. 6. The fit to the MiniBooNE data is more consistent with the values of $F_A(Q^2)$ extracted from pion electroproduction on free nucleons (shown in Fig. 6(b)), than with the values $F_A(Q^2)$ extracted from neutrino data on deuterium (Fig. 6(a)).

TABLE 1. Fits to MiniBooNE neutrino quasielastic scattering data on carbon

Form Factors vector/axial	data set (2D)/(1D)	M_A (GeV)	N normalization	χ^2/NDF model	flux
BBBA07 FA=Dipole	double diff (2D)	1.35 ± 0.02	0.99 ± 0.01	$39.8/135 = 0.30$	nominal
	$d\sigma/dQ^2$ (1D)	1.41 ± 0.04	0.99 ± 0.02	$11.7/15 = 0.78$	nominal
BBC(TE) FA=dipole	double diff (2D)	1.22 ± 0.02	1.01 ± 0.01	$43.6/135 = 0.32$	nominal
	$d\sigma/dQ^2$ (1D)	1.17 ± 0.03	1.05 ± 0.02	$19.7/15 = 1.31$	nominal
BBC(TE) FA=mod. dipole	double diff (2D)	1.17 ± 0.02	1.01 ± 0.01	$35.2/135 = 0.28$	nominal
	$d\sigma/dQ^2$ (1D)	1.11 ± 0.03	1.04 ± 0.02	$19.0/15 = 1.27$	nominal
BBC(TE) FA=mod. dipole	double diff (2D)	1.17 ± 0.02	1.01 ± 0.01	$35.4/135 = 0.26$	Flux A
	$d\sigma/dQ^2$ (1D)	1.10 ± 0.03	1.04 ± 0.02	$17.8/15 = 1.18$	Flux A
BBC(TE) FA=mod. dipole	double diff (2D)	1.17 ± 0.02	1.01 ± 0.01	$38.3/135 = 0.28$	Flux B
	$d\sigma/dQ^2$ (1D)	1.09 ± 0.03	1.04 ± 0.02	$17.7/15 = 1.18$	Flux B

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